



BCATS

BUILDING, CONSTRUCTION
AND ALLIED TRADE SKILLS

Maths processes



Unit Standard 24361 (v3), Level 2

Apply mathematical processes
to BCATS projects

3 CREDITS

BCITO
buildingpeople

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Introduction

This handbook contains a range of mathematical processes used in building and construction industries. We suggest that you:

1. skim through this handbook before you begin your projects
2. go back to the sections that are needed for your projects and work through the mathematical processes in more detail
3. ask your teacher/tutor for help when you need to.

The importance of taking the opportunity to be confident with maths before leaving school cannot be over emphasised. The ability to apply a range of trade calculations will help ensure that timber components are cut to the correct lengths, quantities of materials are correctly ordered, windows and doors are made to the size needed, concrete will harden, paint will dry...the list of why maths is important is endless.

How you will be assessed

You need to choose and apply the correct mathematical processes to at least two projects. You will need to choose and use trigonometry. You will also need to choose and use at least one of

- calculations
- measurement
- geometry.

Your teacher/tutor might give you a work diary and/or templates for cutting lists and order lists to help you record calculations.

If you are completing this unit standard in a maths class, you will be given Knowledge Assessment Sheets to complete. Your teacher/tutor may also give you the opportunity to apply your knowledge to solve problems around the school.

If you are completing this unit standard as you do your BCATS projects, you will be able to apply your maths directly to the projects you're doing. Your teacher/tutor may also give you a Knowledge Assessment Sheet to complete. If you can, take photos of your projects to show how you successfully applied your maths skills.

You need to show your teacher/tutor you can:

- identify what information you need to find
- choose the best mathematical methods for the situation and problem
- correctly use the chosen methods
- accurately present the information and results.

Mathematical signs and symbols

There are a lot of mathematical signs and symbols that you will come across in documents, instructions, and plans and that you might use when you are working on a project.

| Symbol | Meaning |
|----------------|---|
| = | Equal to |
| + | Add |
| ÷ | Divide |
| x | Multiply |
| – | Subtract |
| : | Ratio |
| x^2 | Number squared. This tells you to multiply the number twice. For example 5^2 is 5×5 . |
| $\sqrt{\quad}$ | Square root. To find out what number multiplied by itself has totaled your given number, i.e $\sqrt{25} = 5$ (because $5 \times 5 = 25$). |
| m^2 | Metres squared or square metres. A unit of area measurement calculated by multiplying length x width. |
| m^3 | Metres cubed or cubic metres. A unit of volume measurement calculated by multiplying length x width x depth. |
| π | Pi – the ratio of a circle's circumference to its diameter and is often written as 3.142. |
| % | Percent |
| $\frac{1}{2}$ | A fraction showing 'half'. Fractions show a proportion of a whole and have a numerator (top number) and denominator (bottom number) that is separated by a fraction line. |
| . | Decimal point. Used to separate whole numbers from decimal parts. |

Mathematical signs and symbols

Units of measurement

| Measuring | Metric unit | Shown by | Examples of use |
|-------------|-------------------------|----------------|---|
| Distance | Metre (Lineal meter) | m | Lengths of walls, timber and other materials that are supplied in random or selected sizes and lengths. |
| | Millimetre | mm | |
| Area | Square metre | m ² | Tiles, paving materials, wall linings and flooring. |
| Volume | Cubic metre | m ³ | Stone, soil, sand, concrete. |
| Capacity | Litre | l | Paint, glues, pre-mixed plaster. |
| | Millilitre | ml | |
| Mass | Tonne | t | Stone, bulk material delivery. |
| | Kilogram | kg | Bags of product such as cement, plaster mix. |
| | Gramm | gm | |
| Time | Hour | hr | Curing times, travel time, job completion time, timesheets. |
| | Minutes | min | |
| Temperature | Degree (Celsius) | °C | Conditions for curing, setting, drying etc. |
| Quantity | Number | No. or # | Ties and other fixings. |



This drawing convention defines the **space** that relates to a measurement.

Mathematical signs and symbols

Metric measurements

New Zealand uses the metric measurement system. In the metric measurement system, there is a standard way of adapting the basic units (and their abbreviations) to make/show bigger or smaller units.

The word kilo comes from ancient Greek and means 1,000. When put in front of a basic unit of measurement (such as metre) it shows that it is **1,000 times bigger**.

Kilometre = 1,000 metres

Kilolitre = 1,000 litres

Kilogram = 1,000 grams

The word **milli** comes from Latin and means 1,000th. When put in front of a basic unit of measurement (such as metre) it shows that it is 1,000 times smaller.

Millimetre = 1,000th (or 0.001) of a metre or 1000mm = 1m

Millilitre = 1,000th (or 0.001) of a litre or 1000ml = 1l

Milligram = 1,000th (or 0.001) of a gram or 1000mg = 1g

Did you know?

In building and construction, millimetres (mm) is usually the unit used to measure length because of the accuracy and precision that it provides.



Checking measurements of the project legs' to the nearest mm.

Note:

If you are using plans from the internet, check to ensure they do not have " or ' after the numbers. If they do, the measurements are imperial (inches), not metric (millimetres). You will need to be careful to accurately convert all the measurements into metric.

Building mathematics overview

Smart phones enable instant access to apps, calculators, and conversion tools that make it easy to work out problems. Having a good understanding of the mathematical processes behind any app or tool helps to give you confidence in their results. This is because you will be able to recognise when a result doesn't make sense or doesn't seem quite right. You will then know to check that you entered the information correctly and used the best method to solve the problem.

Applying building measurements and calculations to a project

Calculations are needed throughout all stages of all building and construction projects.

Some of the tasks you will calculations for are:

1. Estimation

Estimating the scope of the job, the quantities of materials, and measurements needed is generally the first step when planning a project and a good reference point after the actual calculations are made.

At every step of the process, predict what you roughly expect the answer to be before you calculate it. This helps you to develop your 'builder's eye' or 'tradie's eye'.

2. Calculating materials

Calculating materials (and overheads) – preparing order lists or cutting lists

Be organised and prepared with the materials needed.

Preparing cutting and order lists helps grow an understanding of the requirements of the whole project (including the costs involved!).

Having the correct quantities of materials on hand before starting the project provides a smoother work flow.

Building mathematics overview

3. Measuring and cutting

The correct use of the units of measurement (and being able to convert units) helps to ensure accuracy. This prevents the need to fix or redo the work, which saves in materials and time.

“Measure twice, cut once”. Always measure straight lines, angles and curves twice before cutting, drilling or installing materials. Measure anything you have cut again once you have cut it. This will give you confidence you have both measured it and cut it correctly.

4. Quality assurance (double checking the calculations)

Always double check your measurements and calculations to check they make sense. This will include measuring other dimensions within the project, such as using Pythagoras Theorem to check if a shape is square (90° corners).



Careful use of measurement and angles can help create projects such as this cupboard and shelving unit made by a Blue Mountain College BCATS student.

Linear measurement systems

Linear measurement includes length, distance, width, height, depth, dimension – what’s the difference?

| | |
|---------------------------|--|
| Length | Length is used for short to mid-range measurements of objects, e.g. the length of timber is 600 millimetres or the length of the wall is 5.6m. |
| Distance | Distance is used for longer measurements or for a measurement between objects, e.g. the distance to town is 2.5km or the distance between the shed and the house needs to be 3m. |
| Width (or breadth) | Width height and depth are also different ways to talk about length. We use these terms when we measure area and volume. |
| Dimension | Dimension is another word for the different measurements as a whole, e.g. the dimensions for a shed includes the length, width and height of the shed. |

Length is usually measured in a straight line, especially shorter lengths which need to be very accurate. Distances may not be straight lines, e.g. a road or a race track.

Units of measurement

In industry, linear measurements use millimetres (mm) and metres (m). Some points to note are:

- 1 metre = 1000 millimetres (1m = 1000mm)
- centimetres are not used because they don’t provide the level of accuracy normally needed
- construction drawings use millimetres unless otherwise stated.

Measurements can be recorded in one of the following ways:

- 1m
- 1.0m
- 1.000m
- 1000mm

Tapes and rulers are the most commonly used tools for measuring lengths.

A tape is more accurate than a ruler when measuring distances over 1m.

A steel or folding ruler is easier to use and more accurate for marking out short lengths of less than 1 metre.

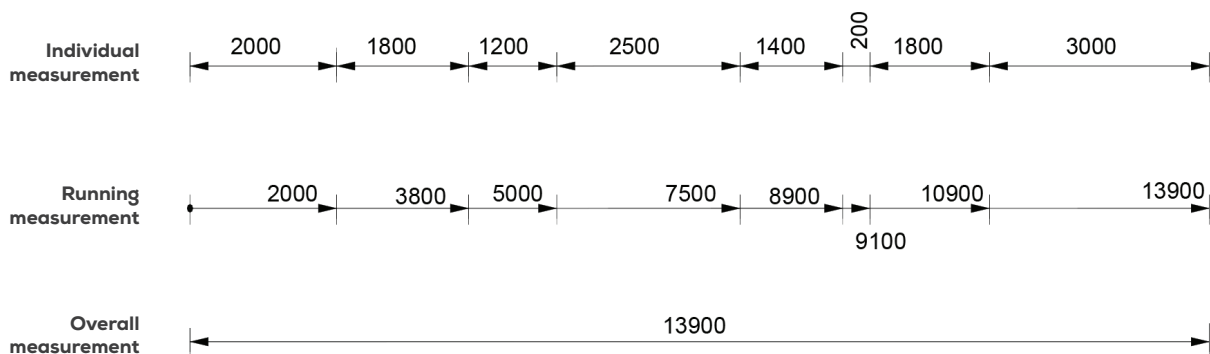
Linear measurement systems

Vernier callipers can offer accurate measurements to very fine tolerances. These are commonly used for precision manufacturing and machining operations. General analog Vernier callipers can measure $1/20$ mm. Digital Vernier callipers can measure with the minimum unit of $1/100$ mm.

Types of measurement

There are three common types of measurement used when constructing buildings and they are each used for different purposes:

- Individual** The distance between two points. Examples of times individual measures will be used are partition walls, opening heights and purlin spaces.
- Running** Is the sum of the individual measures from a given point. Running measures are generally not found on plans. However, using them when setting out the various components of a structure, or a series of objects such as wall frames, studs or trusses results in greater accuracy.
- Overall** The sum/total of the individual measurements or the last of the running measurements. They are often used for setting out the overall size of a structure, double-checking accuracy of individual set-out measurements, calculating area (m^2) or cubic measures (m^3), and for ordering quantities of materials.



The thickness of the materials being used needs to be taken into consideration, such as the width of the timber frames.

- Are the measurements taken from the inside (internal)?
- Are the measurements from the outside (external)?
- Were the measures taken from the centre of the materials?
- Have the materials been included as a separate measure?

Spacings

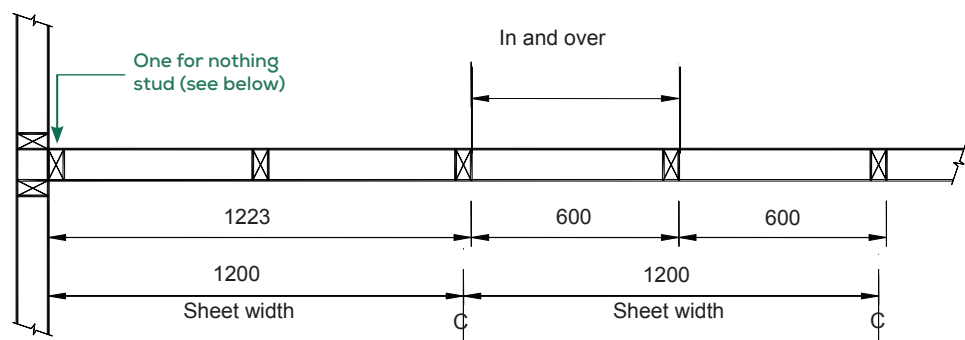
Spacing, or 'setting out', framing members is done when constructing buildings. There are four principles involved in ensuring their equal spacing.

- In-and-over
- One for nothing
- Centre to centre
- In between (sometimes called 'inside')

While each method has its own specific meaning and application, they are all related.

In-and-over to suit sheets (vertical fixing)

The 'in-and-over' method can be used for setting out to suit a standard sheet width, so the joint of the sheet is in the centre of the stud. The first joining stud is measured from the adjacent wall, the width of the sheet plus half the stud thickness. From then on, the in-and-over measurement of 600 applies. This will automatically space the studs so that a 1200mm wide sheet will join on the centre of a stud.



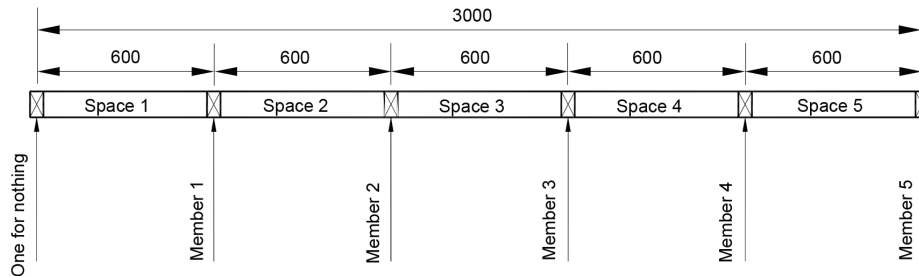
One for nothing

One for nothing means that the first framing timber member in a wall has no span or space associated with it, but still needs to be counted to make sure that that correct number of framing members are ordered.

The sample below shows an example of a section of a wall 3000mm long that requires studs with a maximum spacing of 600mm to be evenly spaced along its length.

In this instance, 'one for nothing' means to include the first stud. The calculation $3000\text{mm} \div 600\text{mm}$. This gives us a result of 5, which is the number of spaces between studs. This does not allow for the first stud in the wall.

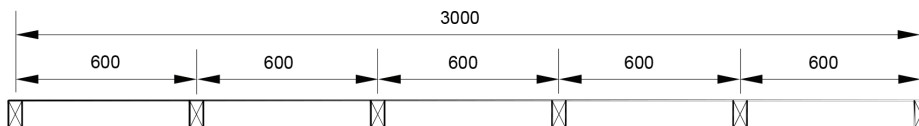
Linear measurement systems



Centre to centre

The centre to centre dimensions are normally recorded in the specification documents or in the working drawings.

The measurement is referring to the distance from the centre of one member to the next member.



In between

This is the distance between the inner faces of two studs. It is the centre-to-centre distance minus the thickness of one stud.

Linear measurement systems

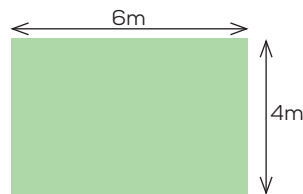
Perimeter

Roughly speaking, the perimeter of an area is the distance around its outside. In a circle, this is called the circumference. If your teacher/tutor told you to run around the boundary of the property, you would be running the perimeter of the property.

The perimeter of a space is the total distance of all its sides. Perimeter is measured in linear metres (m).

Regular shapes

The total number of sides a shape has equals the number of measurements in the calculation.

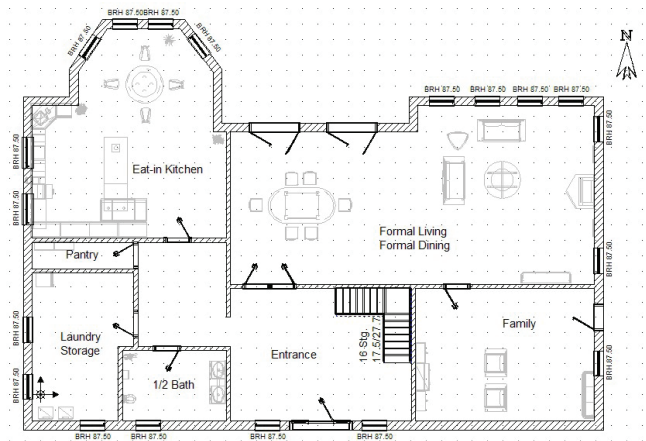


Example:

$$\begin{aligned}\text{Perimeter} &= 4\text{m} + 4\text{m} + 6\text{m} + 6\text{m} \\ &= (4\text{m} \times 2) + (6\text{m} \times 2) \\ &= 20\text{m}\end{aligned}$$

Complex and irregular shapes

The most important thing when calculating the perimeter of a complex shape, is to make sure that all the separate outside lengths are measured and included in the calculation.



Pretty much all the mathematical methods in this handbook were used by Taumarunui High School's Trade Academy students for their Level 3 BCATS playground project.

Calculating area

Area is the amount of space inside a 2D (two dimensional) or flat surface. You are covering the area of the shape if you are painting a wall, paving a courtyard, or buttering bread.

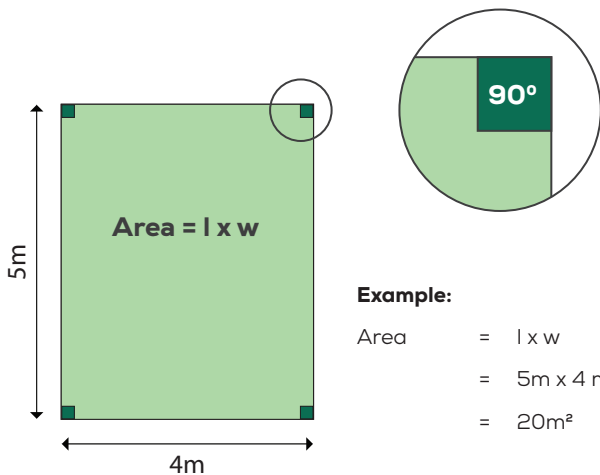
The calculation of areas has many applications in BCATS trades. For example, it is useful for calculating material requirements, areas of building coverage, and for pricing.

Area is measured in square metres (m²). One square metre is the area taken up by a square that is 1 metre long and 1 metre wide.

Areas of straight sided shapes

Squares and Rectangles

The simplest area calculations are for squares and rectangles. Where the shape has four square corners (90°), the method for calculating area is to multiply the length by the height.



Where a shape has square corners, they are usually drawn using this symbol to indicate that the angle is 90°

Example:

$$\begin{aligned} \text{Area} &= l \times w \\ &= 5\text{m} \times 4\text{m} \\ &= 20\text{m}^2 \end{aligned}$$

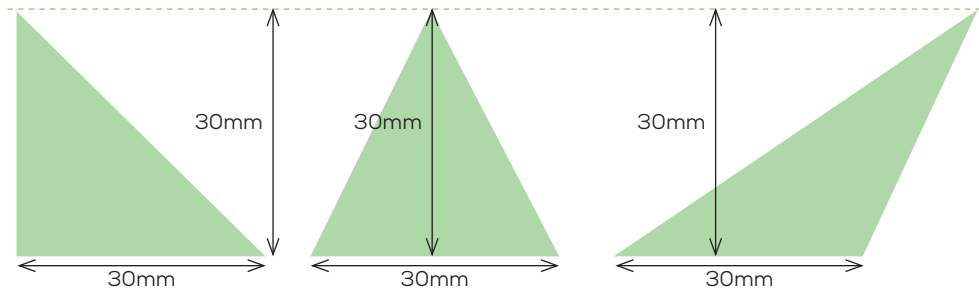
Areas of triangles



It can help to think of a triangle as half of a square or parallelogram. To find the area of a triangle, multiply its height by its width and then divide it by two.

Calculating area

The height of a triangle is measured as a right-angled line from the bottom line to the 'apex' (top point) of the triangle.



The area of the three triangles in the diagram above is the same. Each triangle has a width and height of 30mm.

Example:

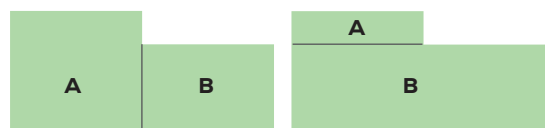
$$\begin{aligned} \text{Area} &= (\text{height} \times \text{width}) \div 2 \\ &= (30 \times 30) \div 2 \\ &= 900 \div 2 \\ &= 450 \text{ mm}^2 \end{aligned}$$

Finding the area of irregular shapes

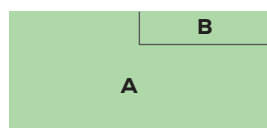
You won't always be working with simple squares or rectangles. If a space has straight sides and right-angled corners, use the same method for finding the area of a simple square.

There are some tricks that make it easier if you are working areas like the ones in the diagram on the right.

The easiest way to work out its area is to split the shape into squares or rectangles joined together.



In these two diagrams, imagine two shapes, work out the area of each, and then add these together to find the total area.



In this diagram, imagine a larger shape (A) and subtract the area of the smaller shape (B) from it to get the answer.

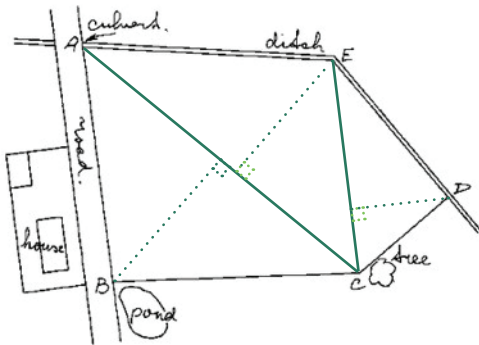
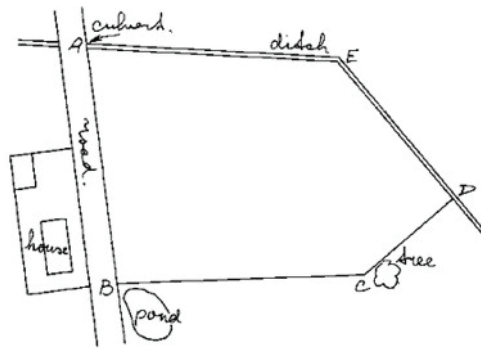
You can split the shapes in any of the ways shown. You will always get the same answer.

Calculating area

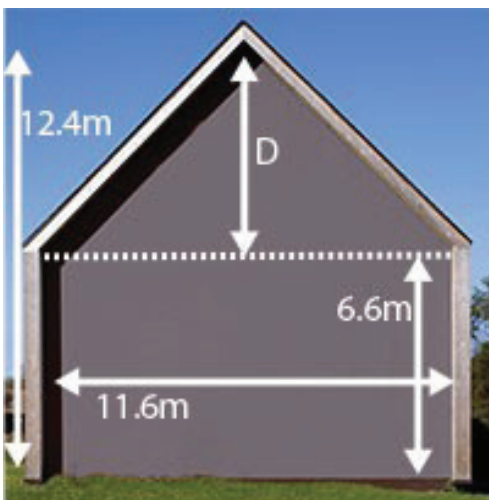
More complex irregular shapes

Even if a space seems to be totally irregular like this example, you can apply the same methods to find out its area.

Start by identifying the regular shapes inside it. Then use the methods you've already learned to work out the area of each of these shapes and add them together.



If you wanted to figure out the area of a wall like the one in the picture below you would be able to do that by imagining the total area split up into two separate shapes, a rectangle and a triangle. This information could then help you to work out, for example, how much paint or replacement cladding is needed.



Example:

Area of wall = rectangle area + triangle area

Rectangle area of wall

$$= 6.6 \times 11.6$$

$$= 76.56 \text{ m}^2$$

Triangular area of wall

$$= (5.8 \times 11.6) \div 2$$

$$= 67.28 \div 2$$

$$= 33.64 \text{ m}^2$$

Area of entire wall

$$= 76.56 + 33.64$$

$$= 110.2 \text{ m}^2$$

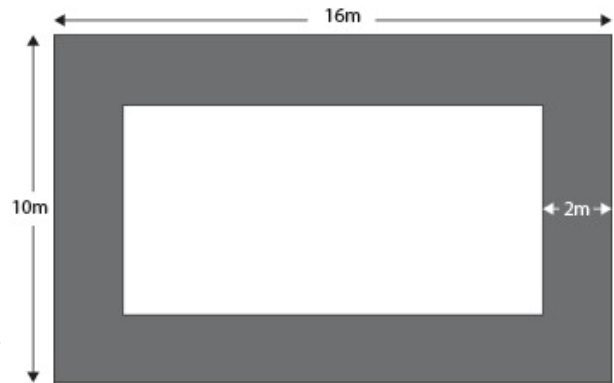
Calculating area

Shapes within shapes

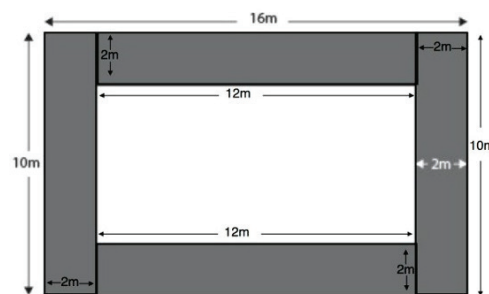
Sometimes you will need to find the area of a border or a shape within another shape.

This example shows a stone path around a lawn. The path is 2m wide.

There are two ways you can work out the area of the path. Both methods will give you the same answer.



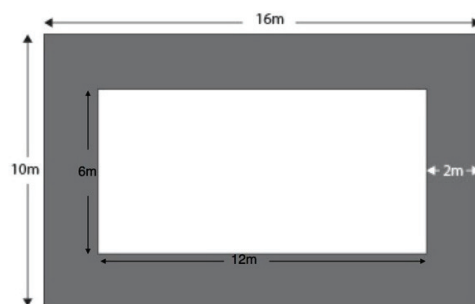
You could imagine the path as four separate rectangles, calculate the area of each rectangle, and add them together to get your total.



Working:

$$\begin{aligned} \text{Area for path is total of each path piece} \\ &= (10 \times 2) + (10 \times 2) + (12 \times 2) + (12 \times 2) \\ &= 20 + 20 + 24 + 24 \\ &= 88 \text{ m}^2 \end{aligned}$$

Or you could work out the total area (measuring the outside length and width), then work out the area of the lawn and subtract this from your total.



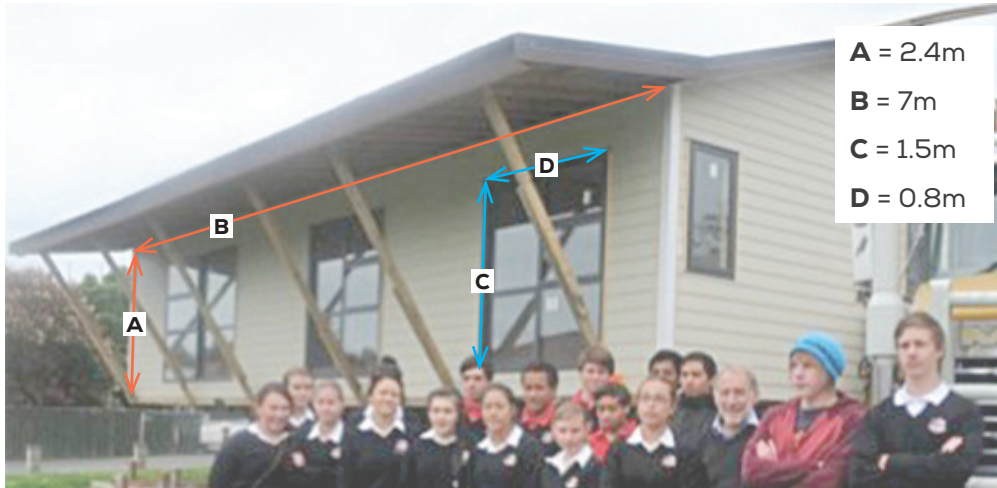
Working:

$$\begin{aligned} \text{Area for path} &= \text{total area} - \text{lawn area} \\ \text{Total area} &= 10 \times 16 \\ &= 160 \text{ m}^2 \\ \text{Lawn area} &= 12 \times 6 \\ &= 72 \text{ m}^2 \\ \text{Path area} &= 160 - 72 \\ &= 88 \text{ m}^2 \end{aligned}$$

Calculating area

More complex shapes within shapes

Unlike most walls, the wall in the previous example didn't include windows or doors. These need to be subtracted from the total area to avoid the expense and waste of materials not needed.



Makoura College students in front of the house they built.

In this example, you need to find the total surface area of the clad part of this house frontage. To calculate this area, you need to do a few different calculations. You need to find the total area of the side of the house and then subtract the area of all the windows.

Working:

$$\begin{aligned}\text{Cladding area} &= \text{area of side of house} - \text{area of windows} \\ &= (A \times B) - (C \times D) \times 3 \quad - \text{there are 3 windows}\end{aligned}$$

$$\begin{aligned}\text{Side of house} &= 2.4 \times 7 \\ &= 16.8 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Windows} &= (1.5 \times 0.8) \times 3 \\ &= 1.2 \times 3 \\ &= 3.6 \text{ m}^2\end{aligned}$$

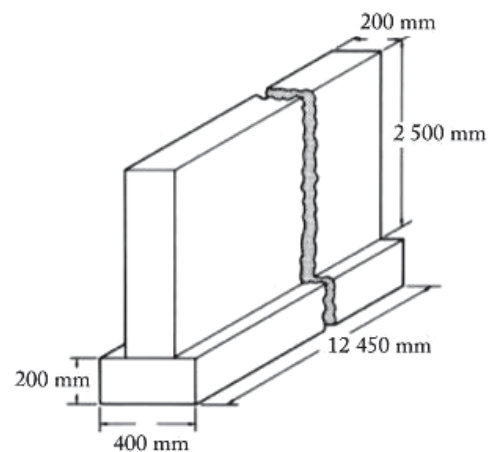
$$\begin{aligned}\text{Area to be clad} &= 16.8 - 3.6 \\ &= 13.2 \text{ m}^2\end{aligned}$$

Calculating volume

Volume is the amount of space inside an object or the amount of space an object occupies (takes up). It is expressed in units cubed, such as cubic metres (m³). Volume calculations are used a lot in BCATS trades.

Volume is calculated by multiplying an object's length by its width and height.

Examples of why calculating volume is important.



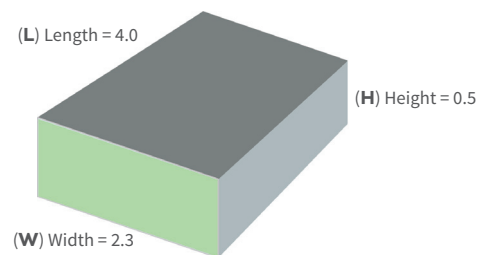
To calculate how many cubic metres of concrete are needed for the foundation wall and footing.



To calculate volume of material from excavation or to calculate amount of material needed for fill.

Cube or cuboid

The volume of a cube (a 3D shape with 6 identical square faces) or cuboid (a 3D shape with 6 rectangular faces) can be calculated in two different ways. Both will give you the same answer.



Method 1:

$$\begin{aligned} \text{Cuboid vol} &= \text{Length} \times \text{Width} \times \text{Height} \\ &= L \times W \times H \\ &= 4 \times 2.3 \times 0.5 \\ &= 4.6 \text{ m}^3 \end{aligned}$$

Method 2:

$$\begin{aligned} \text{Cuboid vol} &= \text{Area of a face} \times \text{Height} \\ \text{Area of face} &= L \times W \\ &= 4 \times 2.3 \\ &= 9.2 \text{ m}^2 \\ \text{Cuboid vol} &= 9.2 \times H \\ &= 9.2 \times 0.5 \\ &= 4.6 \text{ m}^3 \end{aligned}$$

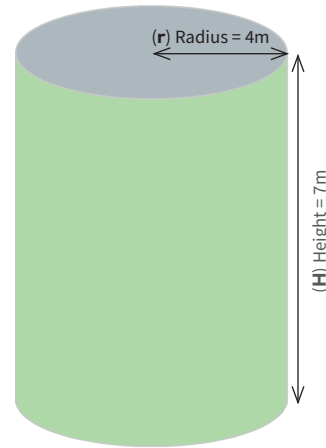
Calculating volume

Cylinder

The easiest way to calculate the volume of a cylinder is similar to method 2 for the volume of a cuboid shown on the previous page. That is, find the **area** of the circular part of the cylinder and multiply it by the height.

So first we have to understand how to calculate the area of a circle.

A circle has dimensions with their own names. These are shown and explained below.



| | |
|----------------------|--|
| Circumference | The circumference is the distance around the edge of a circle |
| Diameter | The diameter of a circle is the measurement that you get if you make a straight line (passing through the centre point) from one side of a circle to the other. |
| Radius | The radius of a circle is the measurement that you get if you make a straight line from a circle's centre point to one of its edges. A circle's diameter is twice the length of its radius (diameter = radius \times 2), which also means that the radius of a circle is half of the diameter (radius = diameter \div 2). |

A 2D diagram of a circle with a grey center. A green arrow around the perimeter is labeled 'circumference'. A double-headed arrow passing through the center from one edge to the other is labeled 'diameter d'. A double-headed arrow from the center to the edge is labeled 'radius r'.

The formula for calculating the area of a circle is πr^2 . This can also be expressed as Pi \times radius squared ($r^2 = \text{radius} \times \text{radius}$).

So to calculate the volume of the cylinder at the top of this page.

Working:

$$\begin{aligned}
 \text{Cylinder volume} &= \text{Area of circular face} \times \text{Height} \\
 \text{Area of circular face} &= \pi r^2 \\
 &= 3.142 \times 4^2 \\
 &= 3.142 \times 16 \\
 &= 50.27 \text{ m}^2 \\
 \text{Cylinder volume} &= 50.27 \times H \\
 &= 50.27 \times 7 \\
 &= 351.89 \text{ m}^3
 \end{aligned}$$

Pi :

Pi is an infinite number but you can round it to 3.142 for your calculations. If you are using a calculator, use the symbol π .

Calculating volume

How much liquid does this tank hold?



Key details:

Diameter of tank = 3.82m

Height of tank = 2.7 m

Important :

1 m³ = 1000 litres

Working:

| | | |
|----------------------------------|---|--|
| Tank (cylinder) volume | = | Area of circular face x Height |
| Area of circular face | = | πr^2 – we need to calculate r (radius) |
| Calculate radius | = | Diameter \div 2 (see previous page) |
| | = | 3.82 \div 2 |
| | = | 1.91 m |
| Calculate area of circular face | = | 3.142 x 1.91 ² |
| | = | 3.142 x 3.6481 |
| | = | 11.46233 m ² |
| Tank volume | = | 11.46233 x H |
| | = | 11.46233 x 2.7 |
| | = | 30.94829 m ³ |
| Convert m ³ to litres | = | m ³ x 1000 |
| | = | 30.94829 x 1000 |
| Capacity of tank | = | 30,948 litres (rounded to whole litres) |

Introduction to trigonometry

Trigonometry, which means literally 'the measurement of triangles', is the branch of mathematics that deals with the relationship and calculation of the unknown sides and angles of a triangle.

Applications of trigonometry can be found extensively in the construction and manufacturing sectors, with the most common relating to calculations involved in the setting out of buildings and roofing calculations. It is also useful in navigation, surveying and other disciplines where distances have to be calculated from the measurement of angles.

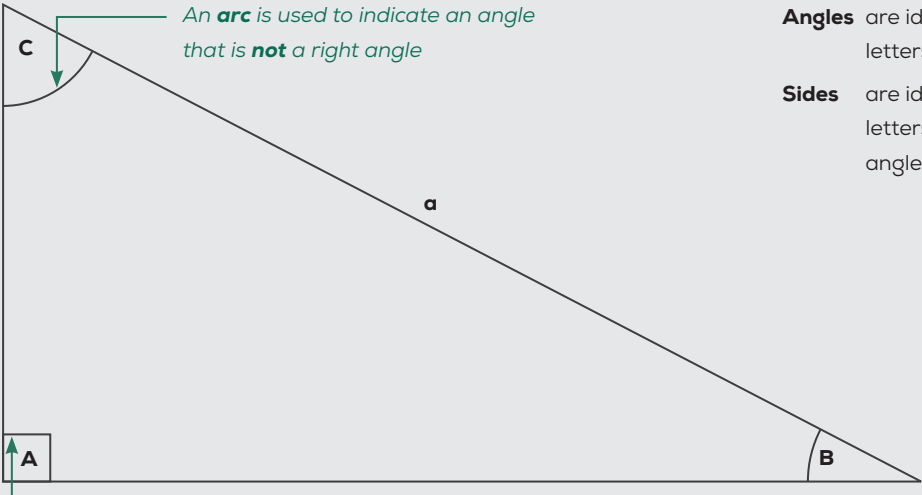
There are two main trigonometry principles used for building and constructions calculations that you need to have a solid understanding of.

- The Triangle Theorem
- The Pythagoras Theorem

Each of these principles has an important role in finding an unknown measurement.

Understanding triangle drawings

Just like we saw with circles (page 19), triangles have their own conventions when it comes to angles and sides. These are detailed below.



An *arc* is used to indicate an angle that is **not** a right angle

A completed **square** indicates a right angle which is always **90°**

Angles are identified by **CAPITAL** letters

Sides are identified with **lowercase** letters and are **opposite** the angle with the same letter

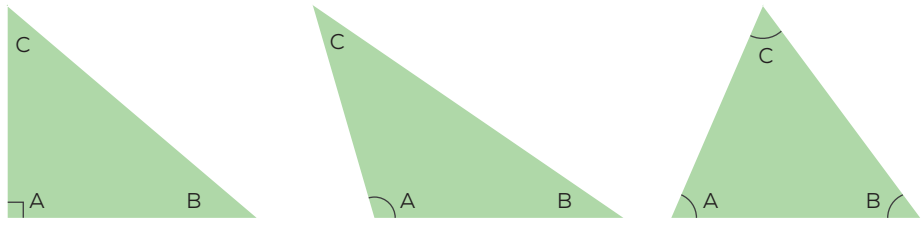
Hypotenuse is the **longest** side in a triangle containing a right angle and is always the side opposite the right angle. This is side **a** in the above triangle.

Adjacent side is the side of a triangle **next to** an angle. Side **c** is the side **adjacent** to angle **B** in the above triangle.

Opposite side is the side directly opposite an angle. Side **b** is the side **opposite** angle **B** in the above triangle.

Introduction to trigonometry

Types of triangle:



Right-angled
Contains a 90° angle

Obtuse-angled
Contains an angle greater than 90°

Acute-angled
All angles less than 90°

Also:
If the hypotenuse squared is longer than the two side lengths squared and added together, then you know that the triangle is obtuse – and that your angle is greater than 90° .
If the hypotenuse squared is shorter than the two side lengths squared and added together, then you know that the triangle is acute – and that your angle is less than 90° .

Triangle theorem

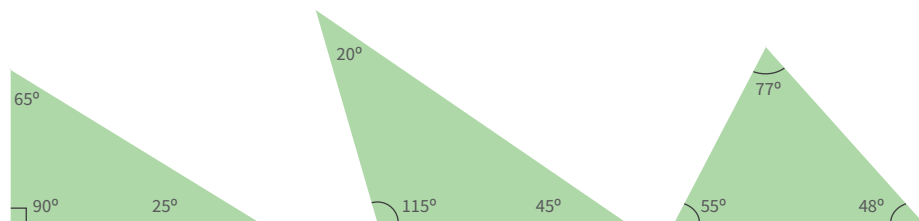
The Triangle Theorem tells us that:

The sum of the three internal angles of a triangle will always add up to 180° .

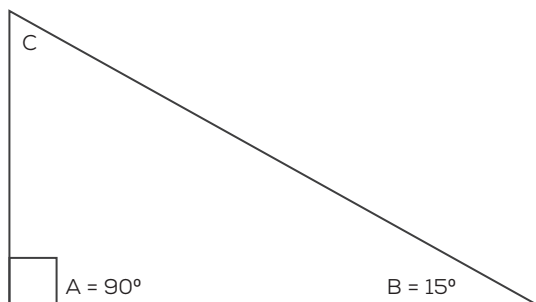
It does not matter if the triangles are right angled, obtuse or acute.

For any triangle:

Angle A + Angle B + Angle C = 180°



If you add up the total of the interior (inside) angles for any triangles above, you will notice that they equal 180° . This allows us to calculate the value of unknown angles: See example below.



Calculate Angle C:

Sum of angles

$$180 = A + B + C$$

$$180 = 90 + 15 + C$$

$$\text{Angle C} = 180 - (90 + 15)$$

$$= 180 - 105$$

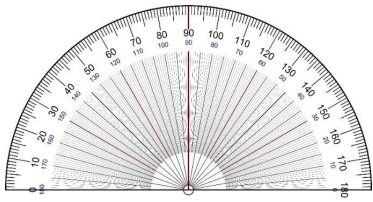
$$= 75^\circ$$

Introduction to trigonometry

Measuring angles

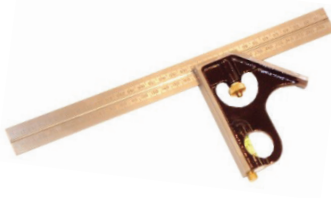
Like most aspects of construction, there are specialist tools for a job. Measuring angles is no exception and the main tools when working with angles are shown below.

Protractor



When you are measuring angles on plans or specifications, it is best to use a protractor.

Combination square



The combination square is best used for marking angles on timber or other materials that are either 45° or 90° .

Sliding bevel



The sliding bevel is a great option for determining and repeating other angle measures on timber or other construction materials.



Imagine the roof as a triangle. The peak is actually 90° , and because the lengths of the roof are different it means that the other two angles are different, but they are both acute angles (less than 90°).

The sides from the base are wider than right angles. They are obtuse angles because they are greater than 90° .

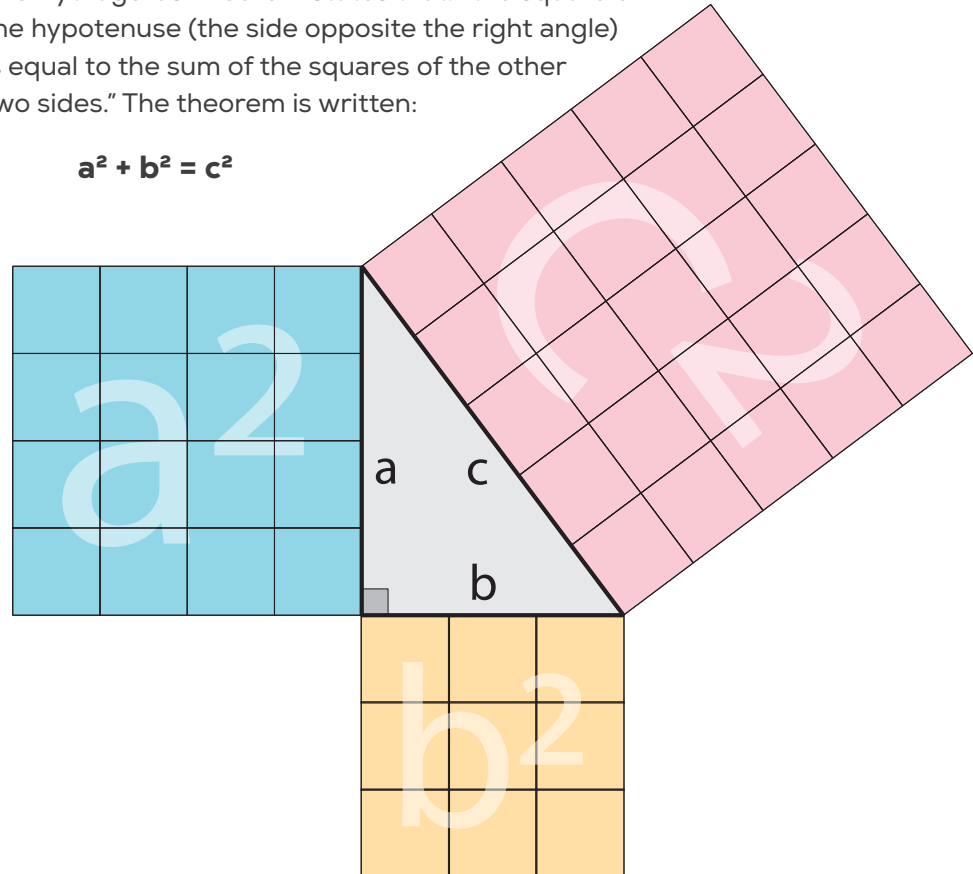


The different angles in this playhouse Awatapu College students constructed add to its attractive finish.

Pythagoras' Theorem

The Pythagoras Theorem states that: "the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides." The theorem is written:

$$a^2 + b^2 = c^2$$



What the $a^2 + b^2 = c^2$ formula means is that regardless of the length of any one side of a right-angled triangle, the relationship between the sides is a constant factor and if the length of two sides is known then the other can be calculated.

In many sketches or drawings of right angled triangles the hypotenuse is often shown as side c , so it is also common to see the formula written as $c^2 = a^2 + b^2$.

Recap:

1. hypotenuse (shown as c in the above diagram) is always the longest side of the triangle and the side opposite the right angle
2. a and b are the other two sides
3. you can only use Pythagoras Theorem (formula) for right angled triangles.

To revise triangle conventions, see the diagram on page 23

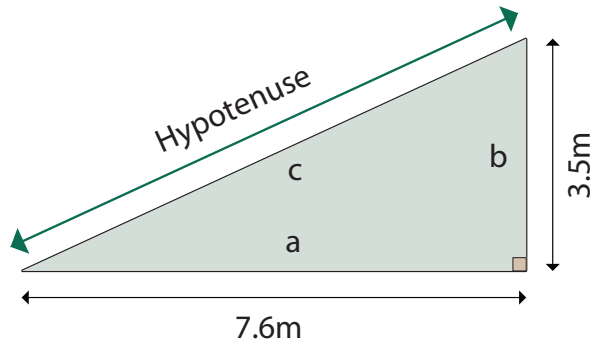
Introduction to trigonometry

Using Pythagoras' Theorem

Calculate the length of the unknown side of the triangle shown in the diagram.

Working:

$$\begin{aligned}\text{Hypotenuse } (c^2) &= a^2 + b^2 \\ &= 7.6^2 + 3.5^2 \\ &= 57.76 + 12.25 \\ c^2 &= 70.01 \\ c &= \sqrt{70.01} \\ &= 8.37\text{m (rounded)}\end{aligned}$$



Remember:

Always be clear about the units being used in a calculation. There is a big difference between 8.37m and 8.37mm.

3:4:5 (Pythagoras Theorem in action)

You might hear the term 3:4:5 being applied to right angled triangles. This is the practical application of the Pythagoras Theorem on the job and you can prove it to yourself by studying the diagram at the top of the previous page.

On the diagram on the previous page, the squares of each side of the triangle have been drawn. You can **count the square of each side**. Side a has 16 squares, side b has 9 squares and side c (the hypotenuse) has 25 squares. $16 + 9 = 25$. ie **$a^2 + b^2 = c^2$**

You aren't limited to using the numbers 3, 4 and 5. You could use multiples of tens, hundreds even thousands – like '30, 40, 50' or '300, 400, 500' or '3000, 4000, 5000'. Or you can use any other multiples of the numbers 3, 4 and 5. Always remember to keep the same formula for each of the numbers in the sequence.

Confirmation of 3:4:5 proportions:

A right angled triangle using the numbers 9, 12, 15, where the numbers 3:4:5 have all been multiplied by 3.

$$\begin{aligned}(9 \times 9) + (12 \times 12) &= (15 \times 15). \\ 81 + 144 &= 225 \\ 225 &= 225 \checkmark\end{aligned}$$

Introduction to trigonometry

More trigonometry

There are also Sine, Cosine and Tangent Ratios. These are sometimes referred to as SOH CAH TOA as abbreviations for the steps involved in calculating them. SOH CAH TOA used to be commonly used in the building and construction industry, such as to calculate lengths and angles of rafters.

If you need to use these mathematical methods, your teacher/tutor can help you. You can also search on the internet for how to use them.

You will need to use a scientific calculator to complete the formulae. If you instead use an IOS (Apple) or Android phone as your calculator, be mindful that the steps are slightly different. You can find how to use these on the internet also.

3:4:5 in action - Example 1

The 3:4:5 Pythagoras Theorem can be used to check that corners are square ie 90° . A masonry example is shown below using the numbers 900mm, 1200mm and 1500mm as their 3:4:5 ratio.



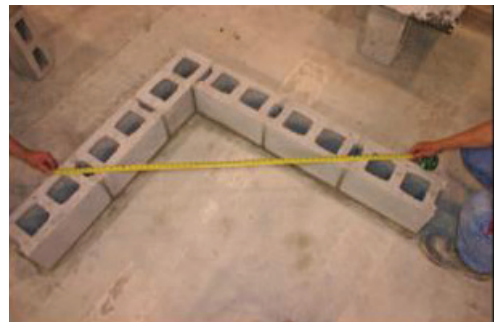
Measure 900mm along one side and mark the brick or block.



Then measure 1200 along the other side and mark the brick or block.



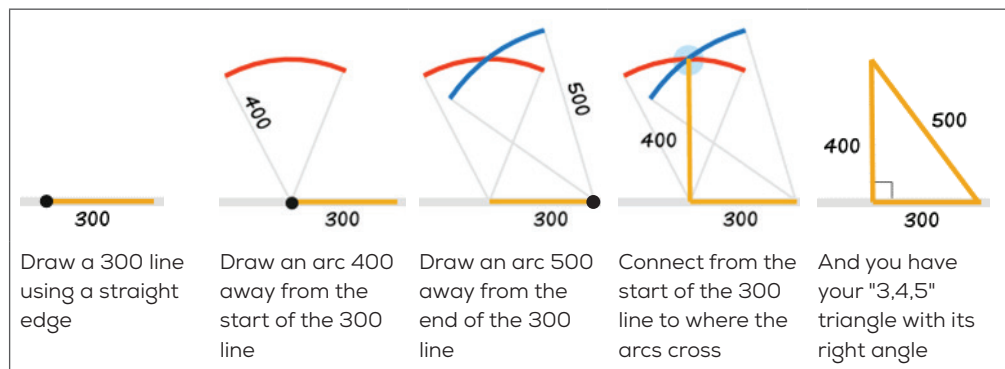
Measure the diagonal.



The diagonal should measure 1500mm from mark to mark

3:4:5 in action - Example 2

You can use the 3:4:5 Pythagoras Theorem to generate a right angle. Let us say you need to mark a right angle coming from a straight line. You decide to use 300, 400 and 500mm lines.



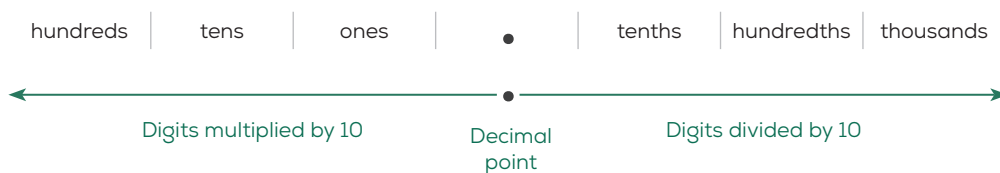
Percentages, fractions, decimals, and ratios

Place values

Depending on the place a digit is positioned in a number tells us a lot about its value.

A good understanding of place values is important when solving construction based problems that involve percentages, fractions, decimals, and ratios. **Place value** is also important when changing units of measures from millimetres to metres and vice versa.

Here is a useful guide to help with understanding the value of different places.



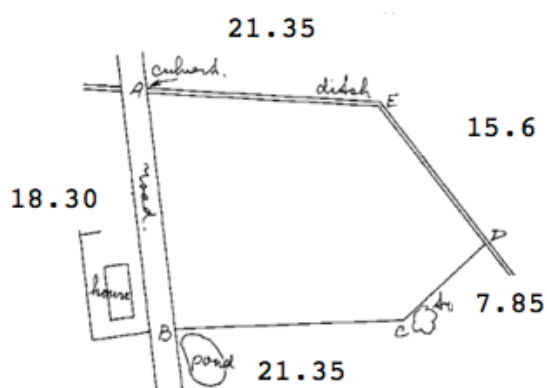
We can understand, or express, an example number **759.35** as follows:

| | | | | | | |
|----------|------|------|---|--------|------------|-------------|
| hundreds | tens | ones | . | tenths | hundredths | thousandths |
| 7 | 5 | 9 | . | 3 | 5 | |

This number has:

- 7 hundreds
- 5 tens
- 9 ones
- 3 tenths
- 5 hundredths

Keeping the digits in each number lined up in its the place value helps to keep the decimal point in the correct place. See the example below.



All figures in m

Example:

Calculate the perimeter

Perimeter = total of all sides

$$\begin{array}{r}
 21.35 \\
 15.6 \\
 7.85 \\
 21.35 \\
 18.30 \\
 \hline
 84.45\text{m}
 \end{array}$$

Rounding numbers

We round numbers to a certain number depending on how much inaccuracy is acceptable. If you need to round materials, always round up.

- When the digit **5, 6, 7, 8,** or **9** appears in the final place, **round up**
- When the digit **1, 2, 3,** or **4** appears in the final place, **round down.**

Examples:

| | |
|-----------------|---------------------------------------|
| Rounding 1.19 | to the nearest tenth gives 1.2 |
| Rounding 1.545 | to the nearest hundredth gives 1.55 |
| Rounding 0.1024 | to the nearest thousandth gives 0.102 |
| Rounding 1.80 | to the nearest one gives 2. |

Fractions

Fractions are used to represent a **part** of an amount that is **less than a whole** number.

A fraction has a numerator and a denominator that shows parts of a whole. Fractions are often used when thinking about dividing a space, adding ingredients together (like half a bucket of something), or planning time on a job.

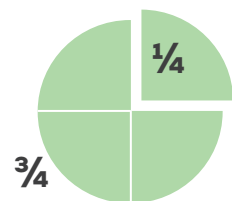
3/4

Numerator → The top number (the numerator) says how many parts you have.

Denominator → The bottom number (the denominator) says how many parts the whole is divided into.

This shape has been split into four parts. This means that the denominator is **4**.

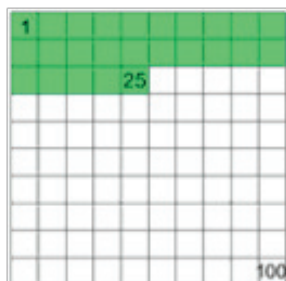
- One part has been removed and can be written as $\frac{1}{4}$.
- Three parts are left, which when written as a fraction, looks like $\frac{3}{4}$



Percentages

The word percent comes from combining the words per and cent (as in century) or hundred. When you see something written down as a percentage (%), that percentage is the amount of 'something' per one hundred units.

Percentages, fractions, decimals, and ratios



There are 100 squares here. 25 of them are shaded. That's 25 per (or out of) 100 which is 25%.

100% of something means 'the whole thing'. This means that you can apply percentages to any number. For example, 100% of 80 is 80; 50% of 80 is 40.

Some percentages worth remembering are:

10% means one tenth

25% means one quarter

50% means one half

75% means three quarters

Percentages as decimals and fractions

Percentages can be written as percentages or as a decimal number or a fraction.

30% is the same as 30 out of 100, which can be written as the fraction $30/100$ or as the decimal 0.3.

Some percentages, decimals, and fractions worth remembering are below.

| Percent | Decimal | Fraction |
|-------------------|---------|----------|
| 1% | 0.01 | $1/100$ |
| 5% | 0.05 | $1/20$ |
| 10% | 0.1 | $1/10$ |
| $12\frac{1}{2}\%$ | 0.125 | $1/8$ |
| 20% | 0.2 | $1/5$ |
| 25% | 0.25 | $1/4$ |
| 33% | 0.33 | $1/3$ |
| 50% | 0.5 | $1/2$ |
| 75% | 0.75 | $3/4$ |
| 80% | 0.8 | $4/5$ |
| 90% | 0.9 | $9/10$ |
| 99% | 0.99 | $99/100$ |
| 100% | 1 | 1 |
| 125% | 1.25 | $5/4$ |
| 150% | 1.5 | $3/2$ |
| 200% | 2 | 2 |

Percentages, fractions, decimals, and ratios

Finding percentages

We will show 3 common methods for calculating using percentage.

Method 1

- First find out what 1% of the total number is. To do this divide the total amount by 100. $\text{Total} \div 100 = 1\%$.
- Next multiply that number (1%) by the percentage you are trying to find. $1\% \times \text{percent wanted}$.

This method is easy to do in your head when on a job site.

Example: When ordering the linear meters of timber for a deck you have to order an additional 20% to cater for off cuts and joins in appropriate places. From reading the plans and job sheet you have calculated that the linear length of timber required is 300m.



The calculations you could follow to ensure the correct amount of timber is ordered are below.

Working:

| | |
|--------------------|-------------------|
| To find 20% of 300 | |
| Find 1% of 300 | = $300 \div 100$ |
| | = 3 |
| Now find 20% | = $1\% \times 20$ |
| | = 3×20 |
| | = 60 |
| Total order | = Required + 20% |
| | = $300 + 60$ |
| | = 360m |

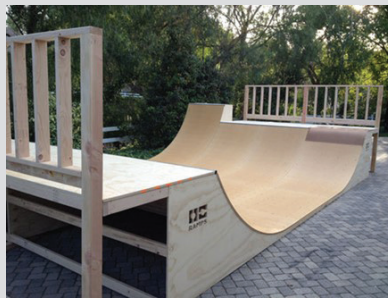
Percentages, fractions, decimals, and ratios

Method 2

- First convert a percentage to a decimal or a fraction. *See the next section in this booklet, page 34.*
- Then multiply the amount by the converted decimal or the fraction.

This method is easy to do in your head when on a job site.

Example: The skate ramp is budgeted to cost \$2300 but you have been extended a 15% discount. That means the discount is subtracted from the budgeted cost.



Working:

$$\begin{aligned} \text{Convert 15\% to a decimal} &= 15 \div 100 \\ &= 0.15 \\ \text{Discount} &= 2300 \times 0.15 \\ &= 345 \\ \text{Final price} &= \text{budget} - \text{discount} \\ &= 2300 - 345 \\ &= \$1955 \end{aligned}$$

Method 3

If you have one, use a calculator.



Calculator steps:

Key presses are: **total x percentage % =**

For the example in Method 1 (previous page)

$$\begin{aligned} &300 \times 20 \% = \\ &= 360\text{m} \end{aligned}$$

Important note:

Whatever method you use for calculations it is important to take care and look at the answer and ask yourself:

- Is the answer reasonable? Is it about the answer that you might expect?
- Do you then need to add or subtract the answer from the total? Is it a reduction, or an increase?

Converting percentages, decimals and fractions

Sometimes you might be given a number written as a fraction and need to change it to a decimal or percentage for your calculations.

Converting a percentage to a decimal and vice versa

Percent to decimal: When changing a percentage to a decimal number, divide the number by 100 and remove the % sign. The easiest way to divide by 100 is to move the decimal point 2 places to the left.

$$75\% \qquad 0.75 \qquad 0.75$$

Decimal to percent: When changing a decimal to a percentage, multiply the number by 100, and add a "%" sign. The easiest way to multiply by 100 is to move the decimal point 2 places to the right.

$$0.125 \qquad 0.125 \qquad 12.5\%$$

Converting decimals to fractions and vice versa

Decimal to a fraction such as, what is 0.73 as a fraction?

- Write the numbers to the right of the decimal point as the numerator (the number above the fraction line).
- Write the number 1 as the denominator (the number below the fraction line), and add as many zeros as the numerator has digits. 73 has two digits so the denominator needs to have two zeros.

$$73 / \leftarrow \text{Numerator}$$

$$73 / 100 \leftarrow \text{Two "0s"}$$

Fraction to a decimal: such as, what is $\frac{73}{100}$ as a decimal?

Simply divide the numerator (top number) by the denominator (bottom number).

That is $73 \div 100 = 0.73$

$$\frac{73}{100} \rightarrow 0.73$$

Decimal

Converting percentages to fractions and vice versa

Percentage to fraction: Converting percentages to fractions is relatively straight forward. Just remember is that a percentage states how much out of 100 - this is easily rewritten as a fraction.

$$73\% \rightarrow \frac{73}{100}$$

73% is the same as 73 out of 100

Fraction to percentage: This requires attention to **place value**, (see previous page) especially tenths, hundredths and thousandths.

1. Ensure that the fraction has a **denominator (bottom number) of 100**. If it does not, write an equivalent fraction that does have 100 as the denominator. (See two examples on the right)

Example 1

$$\frac{3}{5} \quad \text{multiply both numbers by 20} \quad \frac{6}{100}$$

Example 2

$$\frac{6}{15} \quad \text{multiply both numbers by 100} \quad \frac{600}{1500}$$

$$\text{Simplify: divide both numbers by 15} \quad \frac{40}{100}$$

2. The **numerator (top number)** is then written in front of the % (percentage symbol).

$$\frac{40}{100} \quad \text{written as} \quad 40\%$$

Numerator written before the percentage symbol

Calculator method (for any fraction): When using a calculator you are able to convert a fraction to a percentage without having to change the fraction first.

The keystrokes are shown below:

numerator ÷ denominator % =

Example to calculate $\frac{6}{15}$ as a percentage with a calculator:

6 ÷ 15 % =

40%

Percentages, fractions, decimals, and ratios

Simplifying a fraction

Sometimes you will need to simplify a fraction to make it more useable. When simplifying fractions, there are three important things to remember:

1. Changing a fraction occurs by either multiplication or division.
2. Whatever change is decided, apply it to both the numerator and denominator.
3. Look at the starting fraction and ask yourself:
 - Is there a number that both the numerator and denominator can be divided by?
 - Would it be best to multiply by 10 or 100?
 - Would the process to simplify require more than one step?

Here are a few examples to demonstrate how to simplify fractions.

Example 1: Simplify

A fraction like $\frac{12}{48}$ isn't as easy to understand as $\frac{1}{4}$, but they mean the same thing.

- Look at the two numbers in the fraction, 12 and 48. (Both of these numbers can be divided by 12)

$$\frac{12}{48} \rightarrow \text{divide both numerator and denominator by 12} \rightarrow \frac{1}{4}$$

Example 2: Find 0.675 as a fraction

- Write the numbers to the right of the decimal point as the numerator → $\frac{675}{1000}$
- Write the number 1 as the denominator and add as many zeroes as the numerator has digits. → $\frac{675}{1000}$
- Now reduce the fraction. Both the numerator and denominator can be divided by 5.

$$\frac{675}{1000} \rightarrow \text{divide both numerator and denominator by 5} \rightarrow \frac{135}{200}$$

further simplify

$$\frac{135}{200} \rightarrow \text{divide both numerator and denominator by 5} \rightarrow \frac{27}{40}$$

Example 3: Find 3.45 as a fraction

- Write the numbers to the right of the decimal point as the numerator → $3\frac{45}{100}$
- Write the number 1 as the denominator and add as many zeroes as the numerator has digits. → $3\frac{45}{100}$
- Now reduce the fraction. Both the 45 and 100 can be divided by 5

$$3\frac{45}{100} \rightarrow \text{divide both numerator and denominator by 5} \rightarrow 3\frac{9}{20}$$

Ratios

Understanding and applying ratios in construction is important. No one wants plaster that is too wet or dry to apply, cracks in concrete foundations, paint that won't dry, or costly mistakes from misinterpretation of measurements in scaled drawings.

A ratio describes the relationship between two (or more) values. Ratios are represented by a colon (:) that indicates their size in relation to each other. The order of components in a ratio is important.

The ratio 6:1 means that there are six times as many of one unit as there are of the other.



A mix of 6:1 is represented here as six pots of paint to one pot of additive.

Some common ratios in construction include:

- mixing of components to make usable products
- scaled drawings
- spreading rates.

What does 1 part cement and 3 parts sand mean?

Ratios for making concrete mixes often use the word 'part' and means equal sizing.

1 part : **3 parts**
1 part cement : **3 parts builders' mix**

For every 1 part of cement added to the wheelbarrow, add 3 parts of builders' mix. If you need to add 2 parts cement, then you will add 6 parts builders' mix.

Note: You can also think of this as a fraction. For one whole batch of concrete, $\frac{1}{4}$ of the non-water content needs to be cement and $\frac{3}{4}$ builder's mix.



Percentages, fractions, decimals, and ratios

Spreading rate

The ratio below describes the spreading rate of paint.



$$\begin{aligned} 1\text{l} & : & 12\text{m}^2 \\ 1 \text{ litre} & = & 12 \text{ square metres} \end{aligned}$$

This means that it is expected that you will be able to paint 12 square metres for every 1 litre of paint.

Note that paint ratios are sometimes written as 1L/ 12m². This means the same thing as 1l : 12m².

Understanding ratios on a title block

| | | | |
|--|-----------------------|----------------------------------|-----------------|
| Project NIELSEN HOUSE [Building Consent] | | Date 11/03/2019 | Job No. 1554 |
| Drawing Elevations | Scale 1:100 | © Copyright Page 11 | Rev |

In drawing for buildings there is a title block on each page. They are a useful reference point for some key information. This includes the scale of the plans. 'Scale' is the term used for 'ratio' on building plans.

On this drawing the scale is **1:100**. This means that each part of this drawing is 100th of the size of the final project - each real life measurement is 100 times larger than has been drawn.

